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Quiz for October 20, 2004

Recall that \mathbb{Z}_n^\times represents the group of cosets

$$\{m + n\mathbb{Z} \mid m \in \mathbb{Z} \text{ with } m \text{ and } n \text{ relatively prime}\},$$

under the operation of coset multiplication:

$$(m_1 + n\mathbb{Z})(m_2 + n\mathbb{Z}) = m_1m_2 + n\mathbb{Z}.$$

Find all cyclic subgroups of \mathbb{Z}_9^\times .

ANSWER: I am going to write \bar{m} instead of $m + 9\mathbb{Z}$. The elements of \mathbb{Z}_9^\times are $\bar{1}, \bar{2}, \bar{4}, \bar{5}, \bar{7}, \bar{8}$. We see that

$$\begin{aligned}\langle \bar{1} \rangle &= \{\bar{1}\} \\ \langle \bar{2} \rangle &= \langle \bar{5} \rangle = \mathbb{Z}_9^\times \\ \langle \bar{4} \rangle &= \langle \bar{7} \rangle = \{\bar{4}, \bar{7}, \bar{1}\} \\ \langle \bar{8} \rangle &= \{\bar{1}, \bar{8}\}.\end{aligned}$$