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Quiz for October 13, 2011

Let ℓ, m, and n be fixed positive integers and let H be the subgroup

$$H = \{am + bn \mid a, b \in \mathbb{Z}\}$$

of $\mathbb{Z}$. (I believe that H is a subgroup. I do not need to see a proof.) Suppose that H is also equal to $\{c\ell \mid c \in \mathbb{Z}\}$. Prove that ℓ is the greatest common divisor of n and m.

Answer: We see that $m \in H = \{c\ell \mid c \in \mathbb{Z}\}$ so $\ell | m$ and $n \in H = \{c\ell \mid c \in \mathbb{Z}\}$ so $\ell | n$. Thus, ℓ is a common divisor of m and n. We now show that ℓ is the greatest common divisor of m and n. Suppose z is a common divisor of m and n. We must show that $z \le \ell$. If z happens to be negative, then z is certainly less than the positive ℓ; so we need only think about the problem when z is positive. We know that $\ell \in H = \{am + bn \mid a, b \in \mathbb{Z}\}$; so $\ell = am + bn$ for some a and b; but z divides m and z divides n; so z also divides $am + bn = \ell$. Thus, $\ell = \# z$ for some positive integer $\#$ and $z \le \ell$.