PRINT Your Name:________________________________

Quiz for September 15, 2011

Let σ and ρ be the following two elements of $\mathrm{Sym}(\{1,2,3,4\})$. The function ρ has the behavior: $\rho(1) = 2$, $\rho(2) = 3$, $\rho(3) = 4$, $\rho(4) = 1$. The function σ has the following behavior: $\sigma(1) = 1$, $\sigma(2) = 4$, $\sigma(3) = 3$, and $\sigma(4) = 2$. Let H be the smallest subgroup of $\mathrm{Sym}(\{1,2,3,4\})$ which contains σ and ρ.

List the elements in the centralizer of σ in H. Justify your answer.

Recall that if a is an element of the group H, then the centralizer of a in H is $\{h \in H \mid ha = ah\}$. Presumably, you already know that $\rho^4 = \sigma^2 = \mathrm{id}$ and $\sigma\rho = \rho^3\sigma$. You also know that H has 8 distinct elements and each of these elements can be written in the form $\rho^i\sigma^j$ with $i \in \{0,1,2,3\}$ and $j \in \{0,1\}$.

Answer: The centralizer of σ is $\{\mathrm{id}, \sigma, \rho^2, \rho^2\sigma\}$. It is clear that σ and id commute with σ. One computes

$$\sigma\rho^2 = (\sigma\rho)\rho = (\rho^3\sigma)\rho = \rho^3(\sigma\rho) = \rho^3(\rho^3\sigma) = \rho^2\sigma.$$

We have seen that the centralizer of σ is a group. If σ and ρ^2 are in ther centralizer, then $\rho^2\sigma$ is also in the centralizer. We know that $\sigma\rho = \rho^3\sigma$, **and this is different than** $\rho\sigma$; so ρ is not in the centalizer. Again, we use the fact that the centralizer is a group to see that ρ^3, $\rho\sigma$, and $\rho^3\sigma$ are not in the centralizer: if ρ^3, then closure would give $\rho^3\rho^2 = \rho$ is in the centralizer; if $\rho\sigma$ were in the centralizer, then closure would give $(\rho\sigma)\sigma = \rho$ is in the centralizer; if $\rho^3\sigma$ were in the centralizer, then closure would give $(\rho^3\sigma)\sigma = \rho^3$ is in the centralizer.