

## Homework September 29, 2004

1. True or False. If true, prove it. If false, give a counter example. Let  $G$  be a group and let  $H$  be the subset  $H = \{g \in G \mid g^2 = \text{id}\}$ . Then  $H$  is a subgroup of  $G$ .
  
2.
  - (a) Compute the left and right cosets of  $H = \langle \sigma \rangle$  in  $G = D_3$ .
  - (b) Is  $ghg^{-1} \in H$  for all  $g \in G$  and  $h$  in  $H$ , where  $H$  and  $G$  are as given in (a)?
  - (c) Compute the left and right cosets of  $H = \langle \rho \rangle$  in  $G = D_3$ .
  - (d) Is  $ghg^{-1} \in H$  for all  $g \in G$  and  $h$  in  $H$ , where  $H$  and  $G$  are as given in (c)?
  
3.
  - (a) Suppose that  $H$  is a subgroup of the group  $G$  with the property that  $ghg^{-1} \in H$  for all  $g \in G$  and  $h$  in  $H$ . Let  $a$ ,  $b$ , and  $c$  be elements of  $G$  with  $aH = bH$ , prove that  $acH = bcH$ .
  - (b) Suppose that  $H$  is a subgroup of the group  $G$  and that  $a$ ,  $b$ , and  $c$  be elements of  $G$  with  $aH = bH$ . Must  $acH = bcH$ ? Prove or give a counter example.
  
4. Let  $G$  be  $(\mathbb{C} \setminus \{0\}, \times)$ . Describe the left cosets of the subgroup  $H$  in  $G$  where
  - (a)  $H = U_4$
  - (b)  $H = \{ru \mid r \text{ is a positive real number and } u \in U_4\}$ .