My solution to 2 and 3 from Homework November 15, 2004

2. Let G be the group D_3 with elements $\mathrm{id}, \rho, \rho^2, \sigma, \sigma\rho, \sigma\rho^2$. Compute the homomorphism $\Lambda\colon D_3 \to \mathrm{Sym}(D_3)$ as described in 1. It is natural to think of $\mathrm{Sym}(D_3)$ as S_6 where the elements of D_3 are identified with $\{1,2,3,4,5,6\}$ by $1 \leftrightarrow \mathrm{id}$, $2 \leftrightarrow \rho$, $\ldots$, $6 \leftrightarrow \sigma\rho^2$, using the above order for the elements of D_3. For each g in D_3, find the permutaion $\Lambda(g)$ in S_6.

$$
\begin{array}{ccc}
\mathrm{id} & \mapsto & (1) \\
\rho & \mapsto & (1,2,3)(4,6,5) \\
\rho^2 & \mapsto & (1,3,2)(4,5,6) \\
\sigma & \mapsto & (1,4)(2,5)(3,6) \\
\sigma\rho & \mapsto & (15)(2,6)(3,4) \\
\sigma\rho^2 & \mapsto & (1,6)(2,4)(3,5)
\end{array}
$$

3. Let G be the group $\mathbb{Z}_6$ with elements $1,2,3,4,5,0$. Compute the homomorphism $\Lambda\colon \mathbb{Z}_6 \to \mathrm{Sym}(\mathbb{Z}_6)$ as described in 1. It is natural to think of $\mathrm{Sym}(\mathbb{Z}_6)$ as S_6 where the elements of $\mathbb{Z}_6$ are identified with $\{1,2,3,4,5,6\}$ by $1 \leftrightarrow 1$, $2 \leftrightarrow 2$, $\ldots$, $6 \leftrightarrow 0$, using the above order for the elements of $\mathbb{Z}_6$. For each g in $\mathbb{Z}_6$, find the permutaion $\Lambda(g)$ in S_6.

$$
\begin{array}{ccc}
1 & \mapsto & (1,2,3,4,5,6) \\
2 & \mapsto & (1,3,5)(2,4,6) \\
3 & \mapsto & (1,4)(2,5)(3,6) \\
4 & \mapsto & (1,5,3)(2,6,4) \\
5 & \mapsto & (1,6,5,4,3,2) \\
0 & \mapsto & (1)
\end{array}
$$