

Math 546, Final Exam, Summer, 1993

Use your own paper. Each problem is worth 10 points.

1. Let $\sigma = (1, 3, 2)(4, 6, 5)$ and $\tau = (3, 4, 5)$ be elements of S_6 . Write $\tau\sigma\tau^{-1}$ as the product of disjoint cycles.
2. Let a be a fixed element of the group G . Define the function $\varphi: G \rightarrow G$ by $\varphi(g) = aga^{-1}$ for all g in G . Prove that φ is a group ISOMORPHISM.
3. TRUE or FALSE: If H_1 and H_2 are subgroups of the group G , then the UNION $H_1 \cup H_2$ is also a subgroup of G . If the statement is true, then PROVE it. If the statement is false, then give a COUNTEREXAMPLE.
4. Let G be a group and let C be the following subset of G :

$$C = \{c \in G \mid cx = xc \text{ for all } x \in G\}.$$

Prove that C is a subgroup of G .

5. What is the order of the element $(1, 1)$ in the group $\mathbb{Z}_9 \times \mathbb{Z}_6$? Why?
6. Let H be a subgroup of the group G . Suppose that a and b are elements of G with $aH = bH$. Does $a^{-1}H$ HAVE TO EQUAL $b^{-1}H$? If your answer is "yes", then PROVE the statement. If your answer is "no", then give a counterexample.
7. What is the order of the element $(3, 3) + \langle(1, 2)\rangle$ in the group $\frac{\mathbb{Z}_4 \times \mathbb{Z}_8}{\langle(1, 2)\rangle}$? Why?
8. Let $\varphi: \mathbb{Z} \times \mathbb{Z} \rightarrow S_3$ be the function which is defined by

$$\varphi(n, m) = (1, 2)^n(1, 3)^m.$$

Is φ a homomorphism? Why?

9. Let U be the group of complex numbers of modulus one under multiplication, let \mathbb{R} be the group of Real numbers under addition, and let N be the subgroup

$$N = \{2\pi n \mid n \in \mathbb{Z}\}$$

of \mathbb{R} . Prove that $\frac{\mathbb{R}}{N}$ is isomorphic to U .

10. Let $\varphi: G \rightarrow G'$ be a group homomorphism. Prove that

$$\bar{\varphi}: \frac{G}{\ker \varphi} \rightarrow G',$$

given by $\bar{\varphi}(a \cdot \ker \varphi) = \varphi(a)$, is a well-defined function.