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(7) The subgroup $\langle(1,2)\rangle$ consists of

$\{(0,0), (1,2), (2,4), (3,6)\}$

$(3,3) \notin \langle(1,2)\rangle$

$2(3,3) = (2,6) \notin \langle(1,2)\rangle$

$3(3,3) = (1,1) \notin \langle(1,2)\rangle$

$4(3,3) = (0,4) \notin \langle(1,2)\rangle$

$5(3,3) = (3,7) \notin \langle(1,2)\rangle$

$6(3,3) = (2,2) \notin \langle(1,2)\rangle$

$7(3,3) = (1,5) \notin \langle(1,2)\rangle$

$8(3,3) = (0,0) \in \langle(1,2)\rangle$

Thus $(3,3) + \langle(1\,2)\rangle$ has order 8

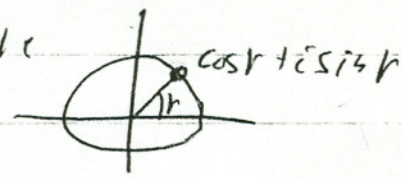
(8) φ is not a homomorphism because

$\varphi[(0,1)+(1,0)] = \varphi[(1,1)] = (12)'(13)' = (132)$

$\varphi(0,1)\cdot\varphi(1,0) = (13)\cdot(12) = (123)$ ⟵ Not equal

(9) Let $\varphi : \mathbb{R} \longrightarrow U$ be the homomorphism

$\varphi(r) = \cos r + i\sin r$. The picture



shows that φ is onto.

the Kernel of φ is N.

So the First Isomorphism Theorem shows $\dfrac{\mathbb{R}}{N} \approx U$.

(10) Suppose $a\cdot\ker\varphi = b\cdot\ker\varphi$. In other words $ab^{-1} \in \ker\varphi$.

we must show $\bar{\varphi}(a\cdot\ker\varphi) = \bar{\varphi}(b\cdot\ker\varphi)$

but $\bar{\varphi}(a\cdot\ker\varphi) = \varphi(a)$

$\bar{\varphi}(b\cdot\ker\varphi) = \varphi(b)$

$ab^{-1} \in \ker\varphi \Rightarrow \underset{e}{\varphi(ab^{-1})} = \varphi(a)\cdot\varphi(b)^{-1}$

Thus $\varphi(a) = \varphi(b)$ ∎