Summer 1993 Exam 2

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① $\tau\sigma\tau^{-1} = (345)(123)(456)(543) = (124)(365)$

② If every permutation in H is even, the the proof is complete. Henceforth, we assume that h is an odd permutation in H.

Let $e_1 \dots e_\ell$ be the collection of even permutations in H

Let $o_1 \dots o_m$ be the collection of odd permutations in H.

Observe that $he_1, \dots, he_\ell$ is a list of ℓ distinct odd permutations in H $\therefore \ell \le m$

Observe that $ho_1, \dots, ho_m$ is a list of m distinct even permutations in H $\therefore m \le \ell$

$\therefore \ell = m$

③ <u>K is Closed</u> take k_1 and $k_2 \in K$. so, there exists h_1 and $h_2 \in H$ with $k_i = a h_i a^{-1}$

$k_1 \cdot k_2 = (a h_1 a^{-1}) \cdot (a h_2 a^{-1}) = a h_1 h_2 a^{-1} \in K$ ✓

<u>Inverses</u> take $k \in K$ so $\exists\ h \in H$ with $k = a h a^{-1}$. observe that $k^{-1} = a h^{-1} a^{-1} \in K$

<u>K is not empty</u> because $e = a e a^{-1} \in K$

④ The set is <u>Not</u> always a group.

<u>Ex</u> $A = \{1, 2, 3\}$ $B = \{1, 2\}$ $b = 1$

$\sigma = \begin{pmatrix} 1 & 2 & 3 \\ 2 & 3 & 1 \end{pmatrix} \in$ the set but $\sigma^{-1} = \begin{pmatrix} 1 & 2 & 3 \\ 3 & 1 & 2 \end{pmatrix} \notin$ set