5. Let $(G, *)$ be an abelian group. Prove that the set

$$S = \{g \in G \mid g * g = \mathrm{id}\}$$

is a subgroup of G.

Closure

Take x and $y \in S$. We know $x*x = id$ and $y*y = id$. We now show $x*y \in S$. Observe that $(x*y)*(x*y) = (x*x)*(y*y) = id$ because G is abelian

id: $id \in S$ because $id * id = id$

Inverses: Take $x \in S$ so $x*x = id$ multiply both sides by $x^{-1} * x^{-1}$ to get $id = x^{-1} * x^{-1}$.

Thus S is a subgroup of G

6. Let G be the group D_3. (a) LIST the elements of the set

$$S = \{g \in G \mid g * g = \mathrm{id}\}.$$

(b) Is S a subgroup of G? Justify your answer to (b).

(a) $id, \sigma, \sigma\rho, \sigma\rho^2$

(b) No. S is not closed, since $\sigma, \sigma\rho \in S$ but $\rho = \sigma(\sigma\rho) \notin S$