3. Define $*$ on $\mathbb{Q}\setminus\{0\}$ by $a*b=\frac{a}{b}$. Is $(\mathbb{Q}\setminus\{0\},*)$ a group? Why or why not?

No. The associative property does not hold.

$$2*(2*2) = 2*\frac{2}{2} = 2*1 = \frac{2}{1} = 2$$

$$(2*2)*2 = \frac{2}{2}*2 = 1*2 = \frac{1}{2}$$

Thus $2*(2*2) \neq (2*2)*2$

4. Recall that $\mathrm{GL}_2(\mathbb{R})$ represents the group of invertible 2×2 matrices with real number entries. The operation in $\mathrm{GL}_2(\mathbb{R})$ is matrix multiplication. The matrix

$$A = \begin{bmatrix} 1 & 0 \\ 2 & 1 \end{bmatrix}$$

is an element of $\mathrm{GL}_2(\mathbb{R})$. What is A's inverse?

A's inverse is $\begin{bmatrix} 1 & 0 \\ -2 & 1 \end{bmatrix}$ because

$$\begin{bmatrix} 1 & 0 \\ 2 & 1 \end{bmatrix}\begin{bmatrix} 1 & 0 \\ -2 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$ and

$$\begin{bmatrix} 1 & 0 \\ -2 & 1 \end{bmatrix}\begin{bmatrix} 1 & 0 \\ 2 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}.$$