 8

17. TRUE or FALSE. (If true, PROVE it. If false, give a COUNTER EXAMPLE.)
All groups of order 5 are isomorphic.

True Let G be a group of order 5. Let $g \in G$ with $g \neq e$. Lagrange's Theorem tells us that $\langle g \rangle = G$. So G is cyclic. We proved in class that all cyclic groups of order n are isomorphic to $\mathbb{Z}_n$. So $G \cong \mathbb{Z}_5$.

18. Prove that the function $\varphi\colon (\mathbb{Z}, +) \to (2\mathbb{Z}, +)$, which is given by $\varphi(n) = 2n$, is a group isomorphism.

$\varphi(n+m) = 2(n+m) = 2n + 2m = \varphi(n) + \varphi(m)$

φ is onto: A typical element of $2\mathbb{Z}$ looks like $2n$ for some $n \in \mathbb{Z}$
But $2n = \varphi(n)$

φ is 1-1: If $\varphi(n) = \varphi(m)$, then $2n = 2m$ in $2\mathbb{Z} \subseteq \mathbb{Q}$ divide by 2 to get $n = m$.