

12. Exhibit two groups of order 25 which are not isomorphic. Explain why the groups are not isomorphic.

\mathbb{Z}_{25} and $\mathbb{Z}_5 \times \mathbb{Z}_5$ are not isomorphic because \mathbb{Z}_{25} has an element $([1]_{25})$ of order 25 but every element in $\mathbb{Z}_5 \times \mathbb{Z}_5$ has order 5 or less.

13. Consider $\varphi: \mathbb{Z}_4 \rightarrow \mathbb{Z}_{12}$, which is given by $\varphi([a]_4) = [a]_{12}$. Is φ a function? Explain.

No $[0]_4 = [4]_4$ But this φ thing carries

equal $\begin{matrix} \nearrow [0]_4 \text{ to } [0]_{12} \\ \searrow [4]_4 \text{ to } [4]_{12} \end{matrix}$ not equal

14. Consider $\varphi: \mathbb{Z}_{12} \rightarrow \mathbb{Z}_4$, which is given by $\varphi([a]_{12}) = [a]_4$. Is φ a function? Explain.

Yes If $[a]_{12} = [b]_{12}$, then $12 \mid b-a$ so $4 \mid b-a$ so $[a]_4 = [b]_4$