

9. What is the order of the element $([4]_6, (12)(34))$ in the group $\mathbb{Z}_6 \times S_4$? Explain

your answer

The order is 6 because $[4]_6$ has order 3 in \mathbb{Z}_6 and $(12)(34)$ has order 2 in S_4 .

10. The subgroup $N = \{\text{id}, (12)(34), (13)(24), (14)(23)\}$ of the group S_4 is normal.

What is the order of the element $N(1234)$ in the group $\frac{S_4}{N}$? Explain your

answer

The order is 2 because $(N(1234))^2 = N(1234)^2 = N(13)(24) = N \cdot \text{id}$

11. Let $(\mathbb{R}^{\text{pos}}, \times)$ represent the group of positive real numbers under multiplication.

Is $(\mathbb{R}^{\text{pos}}, \times)$ isomorphic to $(\mathbb{R}, +)$? If so, exhibit an isomorphism between the two groups. If not, explain why not.

Yes. Consider $\varphi: (\mathbb{R}, +) \rightarrow (\mathbb{R}^{\text{pos}}, \times)$ given by $\varphi(r) = 10^r$. We see that $\varphi(r+s) = 10^{r+s} = 10^r \cdot 10^s = \varphi(r)\varphi(s)$. φ is onto because if $t \in \mathbb{R}^{\text{pos}}$, then $\varphi(\log_{10} t) = 10^{\log_{10} t} = t$. φ is one-to-one because if $\varphi(r) = \varphi(s)$, then $10^r = 10^s$. Take \log_{10} of both sides to see $r = s$.