

3. How many permutations in  $S_7$  have order 6?

6 cycle has order 6. Disjoint product of a 2 cycle and a 3 cycle has order 6. Every element in  $S_7$  of order 6 has been accounted for.

There are  $7 \cdot 5!$  6 cycles in  $S_7$  (7 ways to choose 6 numbers and  $5!$  ways to distribute them: (smallest, 5 choices, 4 choices, 3 choices, 2 choices, 1 choice))

(---)(--) There are  $\binom{7}{5}$  ways to pick 5 numbers. There are  $\binom{5}{3}$  ways to pick 3 (smallest of the 3, 2 choices, 1 choice) (smallest remaining, 1 choice)

$$\binom{7}{5} \binom{5}{3} \cdot 2 = 420$$

$S_7$  has  $840 + 420 = 1260$  elements of order 6

4. Write the cycle (12345) as a product of transpositions.

$$(12345) = (15)(14)(13)(12)$$