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8. TRUE or FALSE. (If true, PROVE it. If false, give a COUNTER EXAMPLE.) If $H$ and $K$ are subgroups of a group $G$, then the intersection $H \cap K$ is also a subgroup of $G$.

True

id $\in H$ because $H$ is a group and id $\in K$ because $K$ is a group so id $\in H \cap K$

closure If $x, y \in H \cap K$, then $x \in H$ and $y \in H$ and $H$ is a group so $xy \in H$

Also $x \in K$ and $y \in K$ and $K$ is a group so $xy \in K$

so $xy \in H \cap K$

inverses Take $x \in H \cap K$. $H$ is a group so $x^{-1} \in H$ and $K$ is a group so $x^{-1} \in K$

so $x^{-1} \in H \cap K$.

9. TRUE or FALSE. (If true, PROVE it. If false, give a COUNTER EXAMPLE.) If $H$ and $K$ are subgroups of a group $G$, then the union $H \cup K$ is also a subgroup of $G$.

False Let $G$ = the subgroup $\{\sigma, \rho^2, \sigma\rho^2, id\}$ of $D_4$

Let $H = \langle \rho^2 \rangle$ and $K = \langle \sigma \rangle$.

We see that $H \cup K = \{\rho^2, \sigma, id\}$ is not a subgroup of $G$

by Lagrange's Theorem since 3 does not divide 4.