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4. State Lagrange's Theorem. If H is a subgroup of the finite group G, then the order of H divides the order of G.

5. True or False (If true, then prove it. If false, then give a counterexample.) If H and K are non-zero subgroups of $(\mathbb{R}, +)$, then the intersection of H and K is non-zero.

False Let $H = \mathbb{Z}$ Let $K = \langle \sqrt{2} \rangle = \{ n\sqrt{2} \mid n \in \mathbb{Z} \}$

Observe that $H \cap K = \{0\}$ because if $r \in H \cap K$, then $r \in \mathbb{Z}$ and $r = n\sqrt{2}$ for some integer n. If $n \neq 0$, then $r = n\sqrt{2}$ tells me that $\frac{r}{n} = \sqrt{2}$. I know that $\sqrt{2}$ is not a rational number so n must be 0; hence $r = 0$.