Math 546, Spring 2004, Exam 4

PRINT Your Name:________________________________

There are 10 problems on 5 pages. The exam is worth 50 points. Each problem is worth 5 points.

I won't grade your exam until Monday. So don't be surprised if I don't e-mail your grade to you until then.

If I know your e-mail address, I will e-mail your grade to you. If I don't already know your e-mail address and you want me to know it, then **send me an e-mail**.

If you would like, I will leave your exam outside my office after I have graded it. (If you like, I will send you an e-mail when I am finished with it.) You may pick it up any time between then and the next class. **Let me know if you are interested.**

I will post the solutions on my website on **Monday**.

1. Write $(1,4)(1,2,3,4,5)(4,6,7)$ as a product of disjoint cycles.

2. Prove that the group of real numbers under addition is isomorphic to the group of positive real numbers under multiplication.

Problems 3, 4, and 5 all refer to the following situation: Let S be a set and let B be a subset of S. Define

$$H = \{\sigma \in \mathrm{Sym}(S) \mid \sigma(b) \in B \text{ for all } b \in B\}.$$

3. Suppose $S = \{1,2,3,4,5\}$ and $B = \{2,3\}$. LIST the elements of H.

4. Return to the general situation as described before problem three. Assume that the set S is finite. Prove that H is a subgroup of $\mathrm{Sym}(S)$.

5. Return to the general situation as described before problem three. Assume that the set S is infinite. Give an example in which H is NOT a subgroup of $\mathrm{Sym}(S)$. Explain your example thoroughly.

6. Let G be a group and a be a fixed element of G. Define $\phi\colon G \to G$ by $\phi(g) = aga^{-1}$ for all $g \in G$. Prove that ϕ is a group isomorphism.

7. Give two non-isomorphic groups of order 36. Explain why the groups are not isomorphic.

8. List the elements of the group $S_3 \times \mathbb{Z}_2$. What is the order of each element?

9. Exhibit an isomorphism $\phi\colon U \to G$, where U is the unit circle group and G is a subgroup of $\mathrm{GL}_2(\mathbb{R})$. Tell me what G is. Tell me what ϕ is. Prove that ϕ is an isomorphism.

10. Exhibit an isomorphism $\phi\colon (\mathbb{R} \setminus \{0\}, \times) \to (\mathbb{R} \setminus \{-2\}, *)$, where $a * b = ab + 2a + 2b + 2$. Tell me what ϕ is and prove that ϕ is an isomorphism.