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12. What is the order of the element $(\overset{u}{\overbrace{\cos\frac{\pi}{3} + i\sin\frac{\pi}{3}}}, \rho)$ in the group $U_6 \times D_4$? Explain your answer.

u has order 6. ρ has order 4. $(u,\rho)^n = (u^n, \rho^n)$. If $(u,\rho)^n = (id, id)$ then 6 must divide n and 4 must divide n. The smallest such n is $n=12$.

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13. Let G be the group $\mathbb{Z}_4 \times \mathbb{Z}_{10}$. Let N be the subgroup $<(2,2)>$ of G. What is the order of the element $(1,2)+N$ in the group $\frac{G}{N}$? Explain your answer.

N consists of $\{(0,0), (2,2), (0,4), (2,6), (0,8), (2,0), (0,2), (2,4), (0,6), (2,8)\}$

$(1,2)+N$ is not the identity element of $\frac{G}{N}$

$(1,2)+N + (1,2)+N = (2,4)+N = (0,0)+N$ is the identity element of $\frac{G}{N}$

So $(1,2)+N$ has order 2 in $\frac{G}{N}$

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14. Let $(\mathbb{R}^{\text{pos}}, \times)$ represent the group of positive real numbers under multiplication. Does $(\mathbb{R}^{\text{pos}}, \times)$ contain any non-cyclic subgroups? If not, explain why not. If so, exhibit such a subgroup and explain why the subgroup is not cyclic.

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The group $\mathbb{R}^{Pos}$ is not cyclic. Cyclic groups are countable or finite. The set $\mathbb{R}^{Pos}$ is not countable.

A different argument is: suppose g generates $\mathbb{R}^{Pos}$. Well $\sqrt{g} \in \mathbb{R}^{Pos}$

so $\sqrt{g} = g^n$ for some integer n

So $g = g^{2n}$ so $g(g^{2n-1} - 1) = 0$

But $g \neq 0$ so $g^{2n-1} = 1$. The only real number with $g^{2n-1} = 1$ is $g = 1$. But 1 does not generate $\mathbb{R}^{Pos}$. This is a contradiction. So $\mathbb{R}^{Pos}$ is not cyclic