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7. Let $\mathbb{Z}$ be the group of integers under addition and let $H$ be the subgroup of even integers. Are the groups $\mathbb{Z}$ and $H$ isomorphic? Explain your answer.

Yes Both groups are infinite and cyclic. (H is generated by 2. We proved that every infinite cyclic group is isomorphic to $\mathbb{Z}$)

8. Are the groups $\mathbb{Z}_{15}$ and $\mathbb{Z}_3 \times \mathbb{Z}_5$ isomorphic? (The operation in each of the groups $\mathbb{Z}_{15}$, $\mathbb{Z}_3$, and $\mathbb{Z}_5$ is addition.) Explain your answer.

Yes $\mathbb{Z}_3 \times \mathbb{Z}_5$ is a cyclic group of order 15. We proved that every cyclic group of order 15 is isomorphic to $\mathbb{Z}_{15}$.

(The group $\mathbb{Z}_3 \times \mathbb{Z}_5$ is generated by $(1,1)$. We see that $10(1,1) = (1,0)$ and $6(1,1) = (0,1)$. So

$$\begin{aligned}(a,b) &= a(1,0) + b(0,1)\\ &= 10a(1,1) + 6b(1,1)\\ &= (10a+6b)(1,1).\end{aligned}$$

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