5. Let a and b be elements of finite order in a group G. LIST two hypothesis so that if a and b satisfy these hypotheses, then the order of ab is equal to the order of a times the order of b. PROVE the result.

Let $\theta(x)$ mean the ord

Hyp 1 $\theta(a)$ and $\theta(b)$ are relatively prime

Hyp 2 $ab = ba$

Proof Let $r = \theta(a)$ and $s = \theta(b)$. We will show that $rs = \theta(ab)$

It is clear that $(ab)^{rs} = (a^r)^s (b^s)^r = id^s id^r = id$

so $\theta(ab) \le rs$. Let $t = \theta(ab)$.

We know that $(ab)^t = id$ so $a^t = b^{-t} \in \langle a \rangle \cap \langle b \rangle$ (since $a^t \in \langle a \rangle$ and $a^t \in \langle b \rangle$)

$\therefore$ $\theta(a^t) \mid \theta(a)$ and $\theta(a^t) \mid \theta(b)$

But $\theta(a)$ and $\theta(b)$ are relatively prime so $\theta(a^t) = 1$ so $a^t = id$

so $r \mid t$

Also $b^t = (a^t)^{-1} = id$ so $s \mid t$. But r and s are relatively prime.

with $r \mid t$ and $s \mid t$; thus, $rs \mid t$

we have r, s, t all positive with $t \le rs$ and $rs \mid t$. It follows

in other words $\theta(a)\theta(b) = \theta(ab)$

6. How many permutations in S_7 have order 3? Explain your answer.

There are $2 \cdot \binom{7}{3}$ 3-cycles in S_7

There are $\binom{7}{6} \cdot 40$ elements of the form (__ __ __) (__ __ __)

5 choices 4 choices 2 choices 1 ch.

Put the smallest number here

Put the smallest remaining # here

The total number of permutations in S_7 of order 3 is

$$2 \cdot \frac{7 \cdot 6 \cdot 5}{3!} + 7 \cdot 40$$

$$= 70 + 280$$

$$= 350$$