546 EX4 2001 SP

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PRINT Your Name:________________________________

There are 10 problems on 5 pages. Each problem is worth 10 points.

1. Define *group isomorphism.*

Let G and G' be groups. A function $\varphi: G \to G'$ is a <u>group isomorphism</u> if φ is one-to-one and onto and $\varphi(g_1 g_2) = \varphi(g_1)\varphi(g_2)$ for all elements g_1 and g_2 in G.

2. Let H be a non-zero subgroup of $(\mathbb{Z}, +)$. Prove that H is a cyclic group. (I want you to write down a complete proof. "We did this in class" is not a satisfactory answer.)

Let h be the smallest positive element of H. I will show that H is generated by h. If m is any element of H, then divide h into m to get $m = ah + b$ for integers a and b with $0 \le b \le h-1$. We see that $m - ah = b \in H$. The choice of h yields that $b = 0$ so $m = ah$ and $H = \langle h \rangle$.