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3. Is the group $(\mathbb{Z}_9^\times, \times)$ a cyclic group? Why or why not?

$\mathbb{Z}_9^\times$ has 6 elements. Also $[2]^2 = [4]$, $[2]^3 = [8]$, $[2]^4 = [7]$

already I see that $\langle [2] \rangle$ has at least 5 elements. Lagrange's Theorem assures me that $\langle [2] \rangle = \mathbb{Z}_9^\times$

Thus $\mathbb{Z}_9^\times$ is a cyclic group.

4. Let A be a set and b be an element of A. Is

$$G = \{\sigma \in \mathrm{Sym}(A) \mid \sigma(b) = b\}$$

always a subgroup of S_A? If your answer is yes, then PROVE the statement. If your answer is no, then give a COUNTEREXAMPLE.

This set G is a group.

closure If σ & τ are in G, the $\sigma \circ \tau \in \mathrm{Sym}(A)$ and $\sigma \circ \tau(b) = \sigma(\tau(b)) = \sigma(b) = b$. It follows that $\sigma \circ \tau \in G$

identity $id \in G$ ✓

inverses Suppose $\sigma \in G$. Then $\sigma^{-1} \in \mathrm{Sym}(A)$ Also $\sigma(b) = b$ s $b = \sigma^{-1}(b)$ Thus $\sigma^{-1} \in G$.

There is no need to check that G is associative because G is a subset of $\mathrm{Sym}(A)$ and composition associates $\mathrm{Sym}(A)$.