4
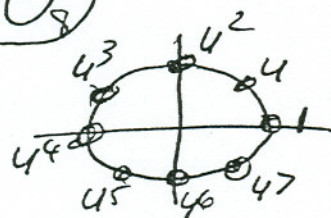
5. Let $U_8 = \{z \in \mathbb{C} \mid z^8 = 1\}$. Which elements in U_8 have the form x^3 for some $x \in U_8$? Explain your answer.

Every element of U_8 has the form X^3 for some $x \in U_8$

Let $u = \cos\frac{2\pi}{8} + i\sin\frac{2\pi}{8} = \frac{\sqrt{2}}{2} + i\frac{\sqrt{2}}{2}$

$1^3 = 1$

$(u)^3 = u^3$

$(u^2)^3 = u^6$

$(u^3)^3 = u$

$(u^4)^3 = u^4$

$(u^5)^3 = u^7$

$(u^6)^3 = u^2$

$(u^7)^3 = u^5$

6. Let $G = D_4$ and $H = \{\text{id}, \sigma\rho^3\}$. Find 4 elements x_1, x_2, x_3, x_4 of G so that G is the disjoint union $[x_1] \cup [x_2] \cup [x_3] \cup [x_4]$, where $[x] = \{y \in G \mid xy^{-1} \in H\}$. Identify x_1, x_2, x_3, x_4 and show the full set $[x_i]$ for each i.

Let $x_1 = id$. $x_2 = \rho$ $x_3 = \sigma\rho$ $x_4 = \sigma\rho^2$

$[x_1] = \{id, \sigma\rho^3\}$

$[x_2] = \{\rho, \sigma\}$

$[x_3] = \{\sigma\rho, \rho^2\}$

$[x_4] = \{\sigma\rho^2, \rho^3\}$