

Please PRINT your name _____

No calculators, cell phones, computers, notes, etc.

Make your work correct, complete and coherent.

Please take a picture of your quiz (for your records) just before you turn the quiz in. I will e-mail your grade and my comments to you. I will keep your quiz.

The quiz is worth 5 points. The solutions will be posted on my website later today.

Quiz 6, March 29, 2023

Find all cyclic subgroups of $\frac{\mathbb{Z}}{\langle 8 \rangle}$. List each subgroup exactly one time.

Answer: The group $\frac{\mathbb{Z}}{\langle 8 \rangle}$ is cyclic of order 8. We proved that every subgroup of a cyclic group is cyclic. We also proved that a cyclic group of order n has exactly one subgroup for each divisor d of n . Indeed, if g has order n , then $\langle g^{n/d} \rangle$ is the subgroup of $\langle g \rangle$ of order d . The group $\frac{\mathbb{Z}}{\langle 8 \rangle}$ has four subgroups. Each of the subgroups is cyclic.

The subgroup generated by $1 + \langle 8 \rangle$ has order 8 and is equal to $\frac{\mathbb{Z}}{\langle 8 \rangle}$.

The subgroup generated by $2 + \langle 8 \rangle$ has order 4 and is equal to

$$\{2 + \langle 8 \rangle, 4 + \langle 8 \rangle, 6 + \langle 8 \rangle, 0 + \langle 8 \rangle\}.$$

The subgroup generated by $4 + \langle 8 \rangle$ has order 2 and is equal to

$$\{4 + \langle 8 \rangle, 0 + \langle 8 \rangle\}.$$

The subgroup generated by $0 + \langle 8 \rangle$ has order 1 and is equal to

$$\{0 + \langle 8 \rangle\}.$$