Please PRINT your name ______________________________________

No calculators, cell phones, computers, notes, etc.

Make your work correct, complete and coherent.

Please take a picture of your quiz (for your records) just before you turn the quiz in. I will e-mail your grade and my comments to you. I will keep your quiz.

The quiz is worth 5 points. The solutions will be posted on my website later today.

Quiz 6, October 27, 2022

Which of the following are homomorphisms? If φ is a homomorphism, then prove it. If φ is not a homomorphism, then give an example which shows that φ does not have the property of being a homomorphism.

(a) $\varphi : (\mathbb{R}\setminus\{0\},\times) \to \mathrm{GL}_2(\mathbb{R})$ defined by $\phi(a) = \begin{bmatrix} a & 0 \\ 0 & 1 \end{bmatrix}$

Answer: This φ is a homomorphism. If a and b are non-zero real numbers, then

$$\varphi(ab) = \begin{bmatrix} ab & 0 \\ 0 & 1 \end{bmatrix}$$

and

$$\varphi(a)\varphi(b) = \begin{bmatrix} a & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} b & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} ab & 0 \\ 0 & 1 \end{bmatrix}.$$

(b) $\varphi : (\mathbb{R},+) \to \mathrm{GL}_2(\mathbb{R})$ defined by $\phi(a) = \begin{bmatrix} 1 & a \\ 0 & 1 \end{bmatrix}$,

Answer: This φ is a homomorphism. If a and b are real numbers, then

$$\varphi(a+b) = \begin{bmatrix} 1 & a+b \\ 0 & 1 \end{bmatrix}$$

and

$$\varphi(a)\varphi(b) = \begin{bmatrix} 1 & a \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & b \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & a+b \\ 0 & 1 \end{bmatrix}.$$

(c) $\varphi : \mathrm{Mat}_{2\times 2}(\mathbb{R}) \to (\mathbb{R},+)$ defined by $\phi\left(\begin{bmatrix} a & b \\ c & d \end{bmatrix}\right) = a$,

Recall that $\mathrm{Mat}_{2\times 2}(\mathbb{R})$ is the Abelian group of 2×2 matrices with real number entries. The operation in $\mathrm{Mat}_{2\times 2}(\mathbb{R})$ is matrix addition.

Answer: This φ is a homomorphism. If

$$\begin{bmatrix} a & b \\ c & d \end{bmatrix} \quad \text{and} \quad \begin{bmatrix} e & f \\ g & h \end{bmatrix}$$

are matrices with real entries, then

$$\varphi\left(\begin{bmatrix} a & b \\ c & d \end{bmatrix} + \begin{bmatrix} e & f \\ g & h \end{bmatrix}\right) = \varphi\left(\begin{bmatrix} a+e & b+f \\ c+g & d+h \end{bmatrix}\right) = a+e$$

$$\varphi\left(\begin{bmatrix} a & b \\ c & d \end{bmatrix}\right) + \varphi\left(\begin{bmatrix} e & f \\ g & h \end{bmatrix}\right) = a + e$$