Please PRINT your name ______________________________________

**No calculators, cell phones, computers, notes, etc.**

Make your work correct, complete and coherent.

Please take a picture of your quiz (for your records) just before you turn the quiz in. I will e-mail your grade and my comments to you. I will keep your quiz.

The quiz is worth 5 points. The solutions will be posted on my website later today.

## Quiz 4, September 29, 2022

Let $(G,*)$ be a group and let $H=\{g\in G \mid g*g*g=\mathrm{id}\}$. Calculate $H$ for $G=D_4$, $G=D_3$, and $G=U_6$. (Recall that $U_6$ is the set of complex numbers which are sixth roots of 1.)

**Answer:**

The identity element $G$ is in $H$ for all $G$. If some element $g$ of $G$ other than the identity element is in $H$, then the order of $g$ must be three (because the $S$ of Homework problem 20 (that is $S=\{n\in\mathbb{Z} \mid g^n=\mathrm{id}\}$) is a subgroup of $\mathbb{Z}$ and the only subgroup of $\mathbb{Z}$ which contains both 2 and 3 is $\mathbb{Z}$).

The group $D_4$ has order 8. we know from Lagrange's Theorem that $D_4$ does not contain any elements of order 3. Thus $H=\{\mathrm{id}\}$ for $D_4$.

In $D_3$ the two rotations have order 3 and the three reflections have order 2; thus the $H$ for $D_3$ is $\langle\rho\rangle$.

Let $\zeta=e^{\frac{2\pi i}{6}}$. In $U_6$, $\zeta$ and $\zeta^5$ have order 6; $\zeta^2$ and $\zeta^4$ have order 3; and $\zeta^3$ has order 2. Thus the $H$ for $U_6$ is $\langle\zeta^2\rangle$.