Please PRINT your name _____

No calculators, cell phones, computers, notes, etc.

Make your work correct, complete and coherent.

Please take a picture of your quiz (for your records) just before you turn the quiz in. I will e-mail your grade and my comments to you. I will keep your quiz.

The quiz is worth 5 points. The solutions will be posted on my website later today.

Quiz 3, September 22 , 2022

Let G be a group, and let $a \in G$. The set $C(a) = \{x \in G \mid xa = ax\}$ of all elements of G that commute with a is called the *centralizer* of a. Prove that $C(a)$ is a subgroup of G.

Answer:

The identity element of G is in $C(a)$.

We show that $C(a)$ is closed. If x and y are elements of $C(a)$, then

$$(xy)a = x(ya) = x(ay) = (xa)y = (ax)y = a(xy);$$

hence $xy \in C(a)$.

We show that if $x \in C(a)$, then the inverse of x in G is also in $C(a)$. If $x \in C(a)$, then $xa = ax$ multiply both sides of the equation on the left by x^{-1} and multiply both sides of the equation on the right by a^{-1} to obtain $ax^{-1} = x^{-1}a$; hence x^{-1} is in $C(a)$.