Please PRINT your name _____

No calculators, cell phones, computers, notes, etc.

Make your work correct, complete and coherent.

Please take a picture of your quiz (for your records) just before you turn the quiz in. I will e-mail your grade and my comments to you. I will keep your quiz.

The quiz is worth 5 points. The solutions will be posted on my website later today.

Quiz 1, January 18, 2023

Let σ and τ be the following elements of S_4:

$$\sigma(1)=3, \quad \sigma(2)=2, \quad \sigma(3)=1, \quad \sigma(4)=4, \quad \text{and}$$
$$\tau(1)=2, \quad \tau(2)=3, \quad \tau(3)=4, \quad \tau(4)=1.$$

Record the multiplication table for the smallest subgroup of S_4 which contains τ and σ. Put your entries in the form $\sigma^i \circ \tau^j$ whenever this makes sense.

Answer: Calculate that $\sigma^2 = \mathrm{id}$, $\tau^4 = \mathrm{id}$ and $(\sigma\tau)^2 = \mathrm{id}$. Calculate that the eight functions $\sigma^i\tau^j$, with $i \in \{0,1\}$ and $j \in \{0,1,2,3\}$ are distinct. It follows that the smallest subgroup of S_4 which contains τ and σ is the set

$$G = \{\sigma^i\tau^j \mid i \in \{0,1\} \text{ and } j \in \{0,1,2,3\}\}.$$

Furthermore, the entire multiplication table for G is

	id	τ	τ^2	τ^3	σ	$\sigma\tau$	$\sigma\tau^2$	$\sigma\tau^3$
id	id	τ	τ^2	τ^3	σ	$\sigma\tau$	$\sigma\tau^2$	$\sigma\tau^3$
τ	τ	τ^2	τ^3	id	$\sigma\tau^3$	σ	$\sigma\tau$	$\sigma\tau^2$
τ^2	τ^2	τ^3	id	τ	$\sigma\tau^2$	$\sigma\tau^3$	σ	$\sigma\tau$
τ^3	τ^3	id	τ	τ^2	$\sigma\tau$	$\sigma\tau^2$	$\sigma\tau^3$	σ
σ	σ	$\sigma\tau$	$\sigma\tau^2$	$\sigma\tau^3$	id	τ	τ^2	τ^3
$\sigma\tau$	$\sigma\tau$	$\sigma\tau^2$	$\sigma\tau^3$	σ	τ^3	id	τ	τ^2
$\sigma\tau^2$	$\sigma\tau^2$	$\sigma\tau^3$	σ	$\sigma\tau$	τ^2	τ^3	id	τ
$\sigma\tau^3$	$\sigma\tau^3$	σ	$\sigma\tau$	$\sigma\tau^2$	τ	τ^2	τ^3	id