Please PRINT your name ______________________________

No calculators, cell phones, computers, notes, etc.

Make your work correct, complete and coherent.

Please take a picture of your quiz (for your records) just before you turn the quiz in. I will e-mail your grade and my comments to you. I will keep your quiz.

The quiz is worth 5 points. The solutions will be posted on my website later today.

Quiz 1, August 25, 2022

Let $S=\mathbb{R}\setminus\{-1\}$. Define $*$ by $a*b=a+b+ab$, for a and b in S. Prove that $(S,*)$ is a group.

Answer:

• We first show that $(S,*)$ is closed. Take a and b in S. It is clear that $a*b$ is a real number. We must show that $a*b=-1$. We agrue by contradiction. If $a*b=-1$, then $a+b+ab=-1$; so $ab+a+b+1=0$. In other words, $(a+1)(b+1)=0$ and $a=-1$ or $b=-1$. Neither of these outcomes is possible. Thus $a*b\neq -1$ and $a*b$ is indeed in S.

• We notice that $*$ commutes!

• Observe that 0 is the identity element of $(S,*)$ because

$$a*0=a+0+a(0)=a.$$

We need not check that $0*a=a$ because we already observed that $a*b=b*a$ for all a and b in S.

• We observe that $*$ associates. Indeed, if a,b,c are in S, then

$$\begin{aligned}a*(b*c)&=a*(b+c+bc)=a+(b+c+bc)+a(b+c+bc)=(a+b+ab)+c+(a+b+ab)c\\&=a*b+c+(a*b)c=(a*b)*c.\end{aligned}$$

• Let a be an element of S. We observe that the inverse of a is $\frac{-a}{a+1}$. First of all, we notice that the proposed inverse is a Real number because $a\neq -1$. We also notice that $\frac{-a}{a+1}\neq -1$ because $0\neq -1$. Thus, the proposed inverse is an actual element of $(S,*)$. Finally, we verify that

$$a*\left(\frac{-a}{a+1}\right)=a+\left(\frac{-a}{a+1}\right)+a\left(\frac{-a}{a+1}\right)=\frac{a(a+1)-a-a^2}{a+1}=\frac{0}{a+1}=0.$$