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14. STATE and PROVE the First Isomorphism Theorem.

Let $\varphi : G \to G'$ be a gp homom.

a) If $N \trianglelefteq G$ and $N < \ker\varphi$, then $\bar{\varphi} : \frac{G}{N} \to G'$ given by $\bar{\varphi}(gN) = \varphi(g)$ is a gp homom.

b) $\bar{\varphi} : \frac{G}{\ker\varphi} \to \operatorname{Im}\varphi$ is a gp iso

Proof a) $\bar{\varphi}$ is welldefined: If $g_1N = g_2N$ then $g_1 = g_2 n$ for some $n \in N$

$\therefore \varphi(g_1) = \varphi(g_2 n) = \varphi(g_2)\varphi(n) \underset{\substack{\uparrow \\ n \in N < \ker\varphi}}{=} \varphi(g_2)$

$\bar{\varphi}$ is a homom: $\bar{\varphi}(g_1N * g_2N) = \bar{\varphi}(g_1g_2N) = \varphi(g_1g_2)$

$\bar{\varphi}(g_1N) * \bar{\varphi}(g_2N) = \varphi(g_1)\varphi(g_2) = \varphi(g_1g_2)$ the same

b) $\bar{\varphi}$ is onto: take $x \in \operatorname{Im}\varphi$ so $x = \varphi(g)$ for some $g \in G$

thus $\bar{\varphi}(g \cdot \ker\varphi) = \varphi(g) = x$

$\bar{\varphi}$ is 1-1: If $\bar{\varphi}(g \cdot \ker\varphi) =$ identity, then

$\varphi(g) =$ identity

so $g \in \ker\varphi$ so $g \cdot \ker\varphi = \ker\varphi$.