66 ~~24~~

12. Give an example of a group G, a subgroup H of G, and elements a, b, and c of G such that $aH = bH$, but $acH \neq bcH$.

$G = S_3$ $\quad H = \langle (12) \rangle = \{(12), (1)\}$ $\quad a = (1)$ $\quad b = (12)$ $\quad c = (13)$

$aH = bH$ ✓

$acH = (13)H = \{(123), (13)\}$ different

$bcH = (132)H = \{(23), (132)\}$

13. FILL IN the blank and then PROVE the resulting sentence. If H is a Normal subgroup of the group G and a, b, and c are elements of G with $aH = bH$, then $acH = bcH$.

we are given $H \triangleleft G$ $\quad a = bh$ for some $h \in H$

we must show $acH = bcH$

$\subseteq$ Take $ach' \in acH$ $\quad ach' = bhch' = bc(c^{-1}hc)h'$

$\in bcH$ because $c^{-1}hc \in H$

$\supseteq$ Take $bch' \in bcH$ $\quad bch' = ah^{-1}ch' = ac(c^{-1}h^{-1}c)h'$

$\in acH$ because $c^{-1}h^{-1}c \in H$