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11. In this problem $\mathbb{R}^+$ represents the set of positive real numbers. Which of the following are groups? Explain each answer.

(a) the set of functions $\{f: \mathbb{R}^+ \to \mathbb{R}^+\}$, under composition of functions,

Not a group. The identity element is $f(x) = x$. There are no inverses. $f(x) = 3$ does not have an inverse.

(b) the set of functions $\{f: \mathbb{R}^+ \to \mathbb{R}^+\}$, under multiplication of functions,

This is a group. $f(x) = 1$ is the identity. The inverse of $f(x)$ is $\frac{1}{f(x)}$. closure and associativity are obvious.

(c) the set of matrices $\{A \in \mathrm{Mat}_{n\times n}(\mathbb{R}) \mid \det A \neq 0\}$ under matrix addition,

Not a group. Not closed.

$$\begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} + \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 2 & 0 \\ 0 & 0 \end{bmatrix}$$

in the set; not in the set

(d) the set of matrices $\{A \in \mathrm{Mat}_{n\times n}(\mathbb{R}) \mid \det A \neq 0\}$ under matrix multiplication.

This is a group.

The identity matrix is the identity element. If $\det A \neq 0$, then A has an inverse. closure and associativity are well known.