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8. All of the following objects are groups. Which of these groups are cyclic groups? Explain each answer.

(a) $\mathbb{Z}_3 \times \mathbb{Z}_3$

Not cyclic  Every element has order 3 or less

$(a,b) + (a,b) + (a,b) = (0,0)$

(b) $\mathbb{Z}_2 \times \mathbb{Z}_3$

cyclic  $(1,1)$ is a generator

(c) The subgroup $\{e^n \mid n \in \mathbb{Z}\}$ of $(\mathbb{R}^*, \times)$.

cyclic  $e$ is a generator

(d) The subgroup $<(1234), (13)(24)>$ of $S_4$.

cyclic  $(1234)$ is a generator. (Notice $(1234)^2 = (13)(24$