

6. TRUE or FALSE. (If true, PROVE it. If false, give a COUNTER EXAMPLE.) Let  $G$  be a group and let  $a$  be a fixed element of  $G$ . If  $\gamma_a: G \rightarrow G$ , is the function which is given by  $\gamma_a(g) = a^{-1}ga$  for all  $g \in G$ , then  $\gamma_a$  is a permutation of the set  $G$ .

True 1-1 suppose  $\gamma_a(g_1) = \gamma_a(g_2)$  then  $a^{-1}g_1a = a^{-1}g_2a$   
Multiply by  $a$  (on the left) and  $a^{-1}$  (on the right)  
to get  $g_1 = g_2$

onto take  $g \in G$  observe that  $\gamma_a(a g a^{-1}) = a^{-1} a g a^{-1} a = g$

7. TRUE or FALSE. (If true, PROVE it. If false, give a COUNTER EXAMPLE.) Let  $G$  be a group and let  $a$  be a fixed element of  $G$ . If  $\rho_a: G \rightarrow G$ , is the function which is given by  $\rho_a(g) = ga$  for all  $g \in G$ , then  $\rho_a$  is a group homomorphism.

False  $\rho_a$  is not a group homomorphism because  $\rho_a(e) = a$  but a group homomorphism always carries the identity to the identity