

PRINT Your Name: _____

There are 14 problems on 8 pages. The exam is worth a total of 100 points. Problems 8 and 11 are each worth 20 points. The other problems are worth 5 points each.

1. DEFINE group. A group is a set G together with an operation $*$ which satisfies:

Closure If a and b are in G , then $a * b \in G$

Associativity If $a, b, c \in G$, then $(a * b) * c = a * (b * c)$

Identity There exists an element $e \in G$ such that $a * e = a = e * a$
For all $a \in G$

Inverse If $a \in G$, then there is an element a^{-1} in G
such that $a * a^{-1} = e = a^{-1} * a$.

2. DEFINE group homomorphism. A group homomorphism is a function $\phi: G \rightarrow G'$ (where G and G' are groups) which satisfies

$$\phi(g_1 g_2) = \phi(g_1) \phi(g_2) \quad \text{for all } g_1, g_2 \in G.$$

3. What is the order of the element $1 - i$ in the group (\mathbb{C}^*, \times) ? Explain your answer.

$\omega = 1 - i$ has infinite order because $|\omega| = \sqrt{2}$

thus $|\omega^n| = (\sqrt{2})^n$ for all n hence ω^n is

never equal to 1 for $1 \leq n$.