


6. TRUE or FALSE. (If true, PROVE it. If false, give a COUNTER EXAMPLE.) If $(G, *)$ is a group, then the function $\phi: G \rightarrow G$, which is given by $\phi(g) = g * g$ is a group homomorphism.

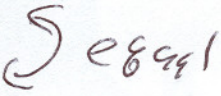
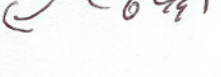
False Take $G = S_3$. $g = (12)$ and $h = (13)$.

We see that $\phi(gh) = \phi((12)(13)) = \phi(132) = (132)^2 = (123)$


But $\phi(g)\phi(h) = (12)^2(13)^2 = \emptyset \cdot \emptyset \leftarrow$  Not equal.

7. TRUE or FALSE. (If true, PROVE it. If false, give a COUNTER EXAMPLE.) Let $a, b,$ and c be elements of a group G and let H be a subgroup of G . If $aH = bH$, then $acH = bcH$.

False Take $G = S_3$. $a = (12)$ $b = \emptyset$ $c = (123)$ $H = \langle a \rangle$

We see that $aH = \{ (12), \emptyset \}$  $bH = \{ (12), \emptyset \}$ 

$acH = (12)(123) \{ (12), \emptyset \} = (23) \{ (12), \emptyset \} = \{ (32), (23) \}$

$bcH = \emptyset (123) \{ (12), \emptyset \} = \{ (13), (123) \}$ 

Not
equal