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6. TRUE or FALSE. (If true, PROVE it. If false, give a COUNTER EXAMPLE.) If $(G, *)$ is a group, then the function $\phi\colon G \to G$, which is given by $\phi(g) = g * g$ is a group homomorphism.

False Take $G = S_3$. $g = (12)$ and $h = (13)$.

We see that $\phi(gh) = \phi((12)(13)) = \phi(132) = (132)^2 = (123)$

But $\phi(g)\phi(h) = (12)^2(13)^2 = id \cdot id$

not equal.

7. TRUE or FALSE. (If true, PROVE it. If false, give a COUNTER EXAMPLE.) Let a, b, and c be elements of a group G and let H be a subgroup of G. If $aH = bH$, then $acH = bcH$.

False Take $G = S_3$ $a = (12)$ $b = id$ $c = (123)$ $H = \langle a \rangle$

We see that $aH = \{(12), id\}$
$bH = \{(12), id\}$ equal

$acH = (12)(123)\{(12), id\} = (23)\{(12), id\} = \{(132), (23)\}$

$bcH = id\,(123)\{(12), id\} = \{(13), (123)\}$

Not equal