

4. TRUE or FALSE. (If true, PROVE it. If false, give a COUNTER EXAMPLE.) Let  $G$  be a group and let  $a$  be a fixed element of  $G$ . If  $\rho_a: G \rightarrow G$ , is the function which is given by  $\rho_a(g) = ga$  for all  $g \in G$ , then  $\rho_a$  is a permutation of the set  $G$ .

True

$\rho_a$  is onto If  $g \in G$ , then  $\rho_a(ga^{-1}) = ga^{-1}a = g$  ✓

$\rho_a$  is 1-1 If  $g$  and  $g'$  with  $\rho_a(g) = \rho_a(g')$  then

$ga = g'a$ . Multiply both sides of the equation on the right by  $a^{-1}$  to see  $gaa^{-1} = g'aa^{-1}$ ; thus

$$g = g'$$

5. TRUE or FALSE. (If true, PROVE it. If false, give a COUNTER EXAMPLE.) Let  $G$  be a group and let  $a$  be a fixed element of  $G$ . If  $\gamma_a: G \rightarrow G$ , is the function which is given by  $\gamma_a(g) = a^{-1}ga$  for all  $g \in G$ , then  $\gamma_a$  is a homomorphism.

True If  $g$  and  $g'$  are in  $G$ , then

$$\gamma_a(gg') = a^{-1}gg'a = a^{-1}gaa^{-1}g'a = \gamma_a(g) \cdot \gamma_a(g')$$