

5. TRUE or FALSE. (If true, PROVE it. If false, give a COUNTER EXAMPLE.)  
If  $A$  is a finite set and  $b$  is an element of  $A$ , then

$$T = \{\sigma \in S_A \mid \sigma(b) = b\}$$

is a group.

True  
Closure Take  $\sigma, \tau \in T$   $\sigma \circ \tau (b) = \sigma(\tau(b)) = \sigma(b) = b$   $\therefore \sigma \circ \tau \in T$   
Inverses If  $\sigma \in T$  then  $\sigma(b) = b$  so  $\sigma^{-1}(b) = \sigma^{-1}(\sigma(b))$   
 so  $b = \sigma^{-1}(b)$

Thus  $\sigma^{-1} \in T$

$T$  is not empty because  $\text{id} \in T$

6. TRUE or FALSE. (If true, PROVE it. If false, give a COUNTER EXAMPLE.)  
If  $A$  is a finite set,  $B$  is a subset of  $A$ , and  $b$  is an element of  $B$ , then

$$T = \{\sigma \in S_A \mid \sigma(b) \in B\}$$

is a group.

False Take  $A = \{1, 2, 3\}$   $B = \{1, 2\}$   $b = 1$

the permutation  $\sigma = (123)$  is in  $T$  since  $\sigma(1) = 2 \in B$   
 but  $\sigma^2 = (132) \notin T$  because  $\sigma^2(1) = 3 \notin B$

Thus  $T$  is not closed.