Fall 1994 Ex 2

PRINT Your Name:________________________________

There are 6 problems on 3 pages. The exam is worth a total of 50 points. Problems 5 and 6 are worth 9 points each. The other problems are worth 8 points each. In this exam, a subgroup H of a group G is called **proper** is $H \subsetneq G$.

1. TRUE or FALSE. (If true, PROVE it. If false, give a COUNTER EXAMPLE.) If every proper subgroup H of a group G is cyclic, then G is cyclic.

False The subgroup $\{id, \sigma, \rho^2, \sigma\rho^2\}$ of D_4 is not cyclic because every element squares to id. But the subgroups of this group all are cyclic because the product of any two of the non-identity elements is the third non-identity element.

2. TRUE or FALSE. (If true, PROVE it. If false, give a COUNTER EXAMPLE.) If G is a cyclic group, then every proper subgroup H of G is cyclic.

True Let $G = \langle g \rangle$. If $H = \{id\}$, then H is cyclic. Hence forth, we assume $\{id\} \subsetneq H$. Let m be the least positive integer with $g^m \in H$. I claim $H = \langle g^m \rangle$. $\supseteq$ is clear

$\subseteq$ If $h \in H$, then $h = g^n$ for some n. Divide m into n to get $n = qm + r$ for integers q and r with $0 \le r \le m-1$. We see that $g^n (g^m)^{-q} \in H$. so $g^r \in H$. The choice of m tells us $r = 0$; hence $n = qm$ and $h = g^{qm} = (g^m)^q \in \langle g^m \rangle$.