$\frac{5}{3}$ (5)

5. Let $\mathbb{R}^*$ represent the set of nonzero real numbers. Define a binary operation $*$ on $\mathbb{R}^*$ by $a * b = b/a$. Is $(\mathbb{R}^*, *)$ a group? If so prove it. If not, show why not.

No. Associativity fails.

$$a*(b*c) = a*(c/b) = (c/b)/a = \frac{c}{ba}$$

not equal

$$(a*b)*c = (b/a)*c = c/(b/a) = \frac{ca}{b}$$

For example Take $c=a=2$ $b=1$

$$(c/b)/a = 1 \qquad c/(b/a) = 4$$

6. Let G be a group. Let

$$H = \{x \in G \mid xy = yx \text{ for all } y \in G\}.$$

Prove that H is a subgroup of G.

$\underline{e \in H}$ $ey = y = ye$ for all $y \in G$

$\underline{\text{Closed}}$ Take x and $z \in H$, Let y be in G $\Big\}$ $(xz)y = xyz = yxz$

$\uparrow$ $z \in H$ $\qquad \uparrow$ $x \in H$

$\therefore (xz)y = y(xz)$ for all $y \in G$

$\therefore xz \in H$

$\underline{\text{inverses}}$ Take $x \in H$ and y in G I know $xy = yx$

Multiply by x^{-1} on left and right $x^{-1}xyx^{-1} = x^{-1}yxx^{-1}$

so $yx^{-1} = x^{-1}y$ for all $y \in G$

~~so $x^{-1}y$~~ $\therefore x^{-1} \in H$.