Math 546, Exam 3, Spring 2010

Write everything on the blank paper provided.

You should KEEP this piece of paper.

If possible: turn the problems in order (use as much paper as necessary), use only one side of each piece of paper, and leave 1 square inch in the upper left hand corner for the staple. If you forget some of these requests, don't worry about it – I will still grade your exam.

The exam is worth 50 points. There are **7** problems.

Write **coherently** in **complete sentences**.

No Calculators or Cell phones.

I will post the solutions later today.

1. (7 points) State Lagrange's Theorem. Use complete sentences. Write everything that is necessary, but nothing extra.

2. (7 points) Define *normal subgroup*. Use complete sentences. Write everything that is necessary for your definition to make sense, but nothing extra.

3. (7 points) Define *group homomorphism*. Use complete sentences. Write everything that is necessary for your definition to make sense, but nothing extra.

4. (8 points) Let H be a normal subgroup of the group G. Suppose that a, b, c, and d are elements of G with $aH = bH$ and $cH = dH$. Prove that $acH = bdH$.

5. (7 points) Give an example of a subgroup H of a group G and elements a, b, c, and d of G with $aH = bH$ and $cH = dH$, but $acH \neq bdH$.

6. (7 points) Write the addition table for $\frac{\mathbb{Z}}{4\mathbb{Z}}$.

7. (7 points) Let $(S, *)$ be the group $(\mathbb{R} \setminus \{-1\}, *)$ with $a * b = ab + a + b$ for all a and b in S. Define the function $\varphi\colon (\mathbb{R} \setminus \{0\}, \times) \to (S, *)$ by $\varphi(r) = r - 1$. Prove that φ is a group homomorphism.