Math 546, Exam 1, Fall, 2022

You should KEEP this piece of paper. Write everything on the **blank paper provided**. Return the problems **in order** (use as much paper as necessary), use **only one side** of each piece of paper. Number your pages and write your name on each page. Take a picture of your exam (for your records) just before you turn the exam in. I will e-mail your grade and my comments to you. I will keep your exam. **Fold your exam in half** before you turn it in.

No calculators, cell phones, computers, notes, etc.

Make your work correct, complete, and coherent.

The exam is worth 50 points. Each problem is worth 10 points.

The solutions will be posted later today.

(1) Suppose H and K are subgroups of the group G. Is the union $H \cup K$ always a subgroup of G? If so, prove the statement. If not, give an example.

(2) Let $(G, *)$ be a group, K be a subgroup of G, g be a fixed element of G with inverse g^{-1}, and $H = \{g * k * g^{-1} \mid k \in K\}$. Prove that H is a group.

(3) Let σ and τ be the following elements of $\mathrm{Sym}(\{1,2,3,4\})$:

$$\sigma(1) = 2, \quad \sigma(2) = 1, \quad \sigma(3) = 3, \quad \sigma(4) = 4,$$

$$\tau(1) = 2, \quad \tau(2) = 3, \quad \tau(3) = 4, \quad \tau(4) = 1.$$

What is the smallest positive integer n with $(\sigma \circ \tau)^n = \mathrm{id}$?

(4) Let σ and τ be the following elements of $\mathrm{Sym}(\{1,2,3,4\})$:

$$\sigma(1) = 3, \quad \sigma(2) = 2, \quad \sigma(3) = 1, \quad \sigma(4) = 4,$$

$$\tau(1) = 1, \quad \tau(2) = 4, \quad \tau(3) = 3, \quad \tau(4) = 2.$$

Let H the smallest subgroup of $\mathrm{Sym}(\{1,2,3,4\})$ which contains σ and τ. What is the multiplication table for H?

(5) Let $(G, *)$ be a group and $H = \{g \in G \mid g * g = \mathrm{id}\}$. Does H have to be a subgroup of G? If so, prove the statement. If not, give an example.