

Solution to the Quiz for June 24, 2003

Let

$$v = \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}, u_1 = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}, u_2 = \begin{bmatrix} -1 \\ 0 \\ 1 \end{bmatrix}, u_3 = \begin{bmatrix} -1 \\ 2 \\ -1 \end{bmatrix}$$

Express v as a linear combination of u_1 , u_2 , and u_3 . **CHECK YOUR ANSWER!** You may do the problem anyway you like, but you might want to notice that u_1 , u_2 , and u_3 are an orthogonal set.

We must find c_1 , c_2 , and c_3 with

$$v = c_1 u_1 + c_2 u_2 + c_3 u_3.$$

Multiply by u_1^T to see that $2 = 3c_1$, by u_2^T to see $-1 = 2c_2$, and by u_3^T to see that $1 = 6c_3$. Thus,

$$\boxed{\begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix} = \frac{2}{3} \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} - \frac{1}{2} \begin{bmatrix} -1 \\ 0 \\ 1 \end{bmatrix} + \frac{1}{6} \begin{bmatrix} -1 \\ 2 \\ -1 \end{bmatrix} .}$$