Solution to the Quiz for April 7, 2003

Find the eigenvalues and eigenvectors of $A = \begin{bmatrix} 1 & -1 \\ 1 & 3 \end{bmatrix}$**.**

The determinant of $A - \lambda I$ is

$$\det \begin{bmatrix} 1-\lambda & -1 \\ 1 & 3-\lambda \end{bmatrix} = (1-\lambda)(3-\lambda) + 1 = \lambda^2 - 4\lambda + 4 = (\lambda - 2)^4.$$

The only eigenvalue for A is $\lambda = 2$. The corresponding eigenspace is the nullspace of

$$A - 2I = \begin{bmatrix} -1 & -1 \\ 1 & 1 \end{bmatrix}.$$

Replace row 2 by row 2 plus row 1.

$$\begin{bmatrix} -1 & -1 \\ 0 & 0 \end{bmatrix}$$

Replace row 1 by minus row 1.

$$\begin{bmatrix} 1 & 1 \\ 0 & 0 \end{bmatrix}$$

The eigenspace of A which belongs to the eigenvalue $\lambda = 2$ is the set of all vectors $\left[\begin{smallmatrix} x_1 \\ x_2 \end{smallmatrix}\right]$ which satisfies

$$\begin{aligned} x_1 &= -x_2 \\ x_2 &= x_2. \end{aligned}$$

The vector $\left[\begin{smallmatrix} 1 \\ -1 \end{smallmatrix}\right]$ is a basis for the eigenspace of A which belongs to $\lambda = 2$.

Check. We see that

$$A \begin{bmatrix} 1 \\ -1 \end{bmatrix} = \begin{bmatrix} 1 & -1 \\ 1 & 3 \end{bmatrix} \begin{bmatrix} 1 \\ -1 \end{bmatrix} = \begin{bmatrix} 2 \\ -2 \end{bmatrix} = 2 \begin{bmatrix} 1 \\ -1 \end{bmatrix}. \checkmark$$