Quiz for March 7, 2003

Problem: Let A be a 5×4 matrix. What is the largest possible rank for A? What is the smallest possible nullity for A? Explain. (Of course, the rank of A is the dimension of the column space of A and the nullity is the dimension of the null space of A.)

Solution: The column space of A is spanned by the 4 columns of A. Some subset of these four columns is a basis for the column space of A (by Theorem 2). Thus the rank of A is at most 4. Theorem 4 says that the nullity of A is equal to the number of columns of A minus the rank of A. The rank of A could be as much as four, so the nullity of A could be as small as zero.