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## Quiz for September 10, 2009

Let $v_{1}, v_{2}, v_{3}$ be linearly dependent vectors in $\mathbb{R}^{m}$. Prove that the vectors $v_{1}, v_{2}, v_{3}, v_{4}$ are linearly dependent for all vectors $v_{4}$ in $\mathbb{R}^{m}$.

ANSWER: Fix an arbitrary vector $v_{4}$ in $\mathbb{R}^{m}$. The first sentence guarantees that there are numbers $a_{1}, a_{2}, a_{3}$, at least one of which is non-zero, with $a_{1} v_{1}+a_{2} v_{2}+a_{3} v_{3}=0$. Thus, we have numbers $a_{1}, a_{2}, a_{3}, 0$, at least one of which is not zero, and $a_{1} v_{1}+a_{2} v_{2}+a_{3} v_{3}+0 v_{4}=0$. We conclude that the vectors $v_{1}, v_{2}, v_{3}, v_{4}$ are linearly dependent.

