PRINT Your Name:__________________________________

Quiz for June 22, 2004

Let W be the subspace of $\mathcal{P}_4$ which is defined as follows: the polynomial $p(x)$ is in W if and only if $p(1)+p(-1)=0$ and $p(2)+p(-2)=0$. Find the dimension of W. Explain.

ANSWER: We find a basis for W. The vector space W is

$$\left\{a_0+a_1x+a_2x^2+a_3x^3+a_4x^4 \,\middle|\, \begin{smallmatrix} a_0+a_1+a_2+a_3+a_4+a_0-a_1+a_2-a_3+a_4=0 \\ a_0+2a_1+4a_2+8a_3+16a_4+a_0-2a_1+4a_2-8a_3+16a_4=0 \end{smallmatrix}\right\}.$$

We solve the equations $Ma=0$ with

$$M=\begin{bmatrix} 2&0&2&0&2\\ 2&0&8&0&32 \end{bmatrix} \quad \text{and} \quad a=\begin{bmatrix} a_0\\ a_1\\ a_2\\ a_3\\ a_4 \end{bmatrix}$$

Apply $R_2 \mapsto R_2-R_1$ to get

$$\begin{bmatrix} 2&0&2&0&2\\ 0&0&6&0&30 \end{bmatrix}.$$

Divide row 1 by 2 and row 2 by 6 to get:

$$\begin{bmatrix} 1&0&1&0&1\\ 0&0&1&0&5 \end{bmatrix}.$$

Apply $R_1 \mapsto R_1-R_2$ to get

$$\begin{bmatrix} 1&0&0&0&-4\\ 0&0&1&0&5 \end{bmatrix}.$$

So, a_0 and a_2 are the dependent variables and a_1, a_3, and a_4 are the free variables. We have

$$\begin{bmatrix} a_0\\ a_1\\ a_2\\ a_3\\ a_4 \end{bmatrix} = a_1\begin{bmatrix} 0\\1\\0\\0\\0 \end{bmatrix} + a_3\begin{bmatrix} 0\\0\\0\\1\\0 \end{bmatrix} + a_4\begin{bmatrix} 4\\0\\-5\\0\\1 \end{bmatrix}.$$

We now know that W is spanned by the polynomials x, x^3, $4-5x^2+x^4$. These polynomials are obviously linearly independent. We conclude that x, x^3, $4-5x^2+x^4$ is a basis for W and W has dimension 3.