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Quiz for June 15, 2004

Let W be a subspace of $\mathbb{R}^n$ and let A be an $m \times n$ matrix. Define V to be the following subset of $\mathbb{R}^m$:

$$V = \{y \in \mathbb{R}^m \mid y = Ax \text{ for some } x \text{ in } W\}.$$

Prove that V is a subspace of $\mathbb{R}^m$.

ANSWER:

The set V is closed under addition: Take arbitrary elements y_1 and y_2 of V. The definition of V tells us that there exist x_1 and x_2 in W with $y_1 = Ax_1$ and $y_2 = Ax_2$, We see that $y_1 + y_2 = Ax_1 + Ax_2 = A(x_1 + x_2)$. We know that $x_1 + x_2$ is in W, because W is a vector space. Thus, $y_1 + y_2$ is in V.

The set V is closed under scalar multiplication: Keep the arbitrary vector $y_1 \in V$ from above. Let r be an arbitrary real number. We see that $ry_1 = rAx_1 = A(rx_1)$. The vector rx_1 is in W, because W is a vector space; and therefore, ry_1 is in V.

The set V contains the zero vector: The zero vector 0 of $\mathbb{R}^n$ is in the vector space W; therefore $0 = A0$ is in V.