## Quiz for June 14, 2005

Let $A$ be an $m \times r$ matrix and $B$ be an $r \times n$ matrix.
(a) Prove that the null space of $B$ is contained in the null space of $A B$.
(b) Prove that the column space of $A B$ is contained in the column space of A.

ANSWER:
(a) Let $x$ be a vector in the null space of $B$. So, $B x=0$. Multiply by $A$ to see $A B x=A 0=0$. Conclude that $x$ is in the null space of $A B$.
(b) Let $y$ be a vector in the column space of $A B$. So, $y=A B x$ for some vector $x$. Thus, $y=A(B x)$, and $B x$ is a column vector. It follows that $y$ is in the column space of $A$.

