PRINT Your Name:________________________________

**Quiz for February 1, 2011**

Let $A$ and $B$ be $n \times n$ matrices. How is $(AB)^{\mathrm{T}}$ related to the product of $A^{\mathrm{T}}$ and $B^{\mathrm{T}}$ ? Prove that your answer is correct.

**ANSWER:** We prove that $(AB)^{\mathrm{T}} = B^{\mathrm{T}}A^{\mathrm{T}}$ . The matrices $(AB)^{\mathrm{T}}$ and $B^{\mathrm{T}}A^{\mathrm{T}}$ both are $n \times n$ matrices. We prove that the corresponding entries are equal. Let $A_{i,j}$ be the entry of $A$ in row $i$ and column $j$ . The entry in row $r$ column $c$ of $(AB)^{\mathrm{T}}$ is

$$[(AB)^{\mathrm{T}}]_{r,c} = (AB)_{c,r} = \sum_{j=1}^{n} A_{c,j}B_{j,r} = \sum_{j=1}^{n} B_{j,r}A_{c,j}$$

$$= \sum_{j=1}^{n} (B^{\mathrm{T}})_{r,j}(A^{\mathrm{T}})_{j,c} = [(B^{\mathrm{T}})(A^{\mathrm{T}})]_{r,c}$$

and this is the entry in row $r$ and column $c$ of $(B^{\mathrm{T}})(A^{\mathrm{T}})$ . The first equality is the definition of transpose. The second equality is the definition of matrix product. The symbols $A_{c,j}$ and $B_{j,r}$ represent numbers and numbers commute under multiplication. This explains the third equality. The fourth equlaity is the definition of transpose, again. The fifth equality is the definition of matrix multiplication, again.